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SOLVING ASSIGNMENT PROBLEM WITH SEVERAL OBJECTIVE FUNCTIONS USING FUZZY SET THEORY

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ABSTRACT:

The A.P. (Assignment Problem) originates from the basic classical problems where the objective is to find the optimum assignment of a number of tasks to an equal number of workers at a minimum time or minimum cost. The several objective assignment problems refer to vector minimum linear programming problems with a special class. Optimization under a fuzzy environment is called fuzzy optimization. Fuzzy several objective linear programming is one of the most frequently applied in fuzzy decision making techniques. Although, it has been investigated and expanded for more than decades by many statisticians and engineers and from the varies point of view, it is still useful to develop new model in order to better result the real world problems within framework of fuzzy several objective linear programming. Fuzzy several objective linear programming problems have its vast applications and scope in the field of new science, technology and engineering. In this paper, assignment problem in which both technical coefficient and available resources are fuzzy with linear and non-linear membership functions was studied and a real approach was proposed to solve the above problem using the techniques proposed by various engineers. In this paper, we use a special type of linear and non-linear membership functions to solve the several objective assignment problems. It gives an optimal compromise solution. The result is compared with linear membership function with non-linear membership functions. Numerical example has been taken to illustrate the solution procedure.

Keywords: - Assignment Problem, several criteria decision making, linear membership function, Non-linear membership function.

INTRODUCTION :

The A.P. (Assignment Problem) is one of the most-studied, well-known and very important problems in mathematical and engineering programming in which our objective is to assign a number of tasks to an equal number of workers so as to minimize the total assignment time or to minimize the total consumed cost for execution of all the tasks. Hence assignment problem can be viewed as a balanced transportation problem, in which all supplies and demands equal to Unity, and the number of rows and columns in the matrix are identical. Hence, Ravindran et al. [6] can be applied the transportation simplex method to solve the assignment problems. However, as an assignment problem is highly degenerate it will be inefficient and not recommended to attempt to solve it by simplex method. Second technique called Hungarian method is commonly employed to solve the minimizing assignment problem by Ravindran et al[7]. Geetha et al.[3] first develop the cost-time minimizing assignment as the problem of multicriteria. Bit et al.[1] use the fuzzy programming technique with linear membership function to solve the multi-objective transportation problem. Tsai et al.[10] used solution for balanced several objective decision making problem associated with time, cost and quality by fuzzy approach. The Linear Interactive and Discrete Optimization (LINDO) Schrage is used for calculation of problem [8], Liebman[5] and TORA packages Taha[9] , General Interactive Optimizer(GINO) as well as

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many other academic and commercial packages are beneficial to find the solution of the transportation problem and assignment problem. Zadeh[12] firstly develop the concept of fuzzy set theory. Then, Zimmermann develop the [13] suitable membership functions to solve linear programming problem with multi objective functions. His result gives optimum solutions showed that solutions obtained by fuzzy linear programming are always efficient. Leberling[4] used a special-type non-linear membership functions for the vector maximum linear programming problem. He showed that solutions obtained by fuzzy linear programming with this type of non-linear membership functions are always efficient. Verma et al.[11] used the fuzzy programming technique with some non-linear (hyperbolic and exponential) membership functions to solve a multi-objective transportation problem are always efficient. Dhingra et al.[2] defined other types of the non-linear membership functions and applied them to an optimal design problem.

In the multi-objective assignment problem, only the objectives are considered as fuzzy. We apply the fuzzy approach with linear and some non-linear membership functions to solve a multi-objective assignment problem as a vector minimum problem.

ASSUMPTIONS AND NOTATIONS :

The notations and assumptions are used in for the proposed model:

i) There are n tasks in a factory and the factory has n workers to process the jobs tasks.

ii) Each task can be associated with one and only one machine.

iii) $C_{ij} \geq 0$ be the execution cost, time etc which is incurred when a job i(i=1,2,…,n) is processed by the machine j(j=1,2,…,n).

iv) The crisp number X_{ij} denotes that the i^{th} task is assigned to the j^{th} work.

v) Each machine can perform each task but with varying degree of efficiency.

MATHEMATICAL FORMULATION :

A several objective assignment problem may be stated mathematically as:

$$\text{Minimize } Z_k = \left\{ \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}^{k} X_{ij} \right. \quad , k = 1,2,\dots,K$$

or

$$\text{Minimize } Z_k = \begin{cases} \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}^{1} X_{ij} \\ \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}^{2} X_{ij} \\ . \\ . \\ . \\ \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}^{k} X_{ij} \end{cases} \qquad (1)$$

subject to

$$\sum_{i=1}^{n} X_{ij} = 1, \qquad j=1,2,\dots,n \qquad (2)$$

$$\sum_{j=1}^{n} X_{ij} = 1, \qquad i=1,2,\dots,n \qquad (3)$$

$$X_{ij} = \begin{cases} 1, \text{ if the } i^{th} \text{ job is assigned to the } j^{th} \text{ machine} \\ 0, \text{ if the } i^{th} \text{ job is not assigned to the } j^{th} \text{ machine} \end{cases} \qquad (4)$$

The constraint in equation no. (2) Ensures that only one task is assigned to one work while the constraint in equation no. (3) Ensures that only one machine should be assigned to one task. And the subscript on Z_k and superscript on c_{ij}^{k} denote the k^{th} penalty criterion.

FUZZY APPROACH FOR THE MULTI-OBJECTIVE ASSIGNMENT PROBLEM :

The several objective assignment problems can be considered as a minimum vector problem. The step first is to assign, for each objective, two values U_k and L_k as upper and lower bounds for the objective function Z_k:

L_k=Aspired level of achievement for objective k, U_k= Highest acceptable level of achievement for objective k and d_k= U_k-L_k the degradation allowance for objective k.

Once the levels of aspiration and degradation for each objective have been specified, then we have formed the fuzzy model. Our second step is to transform the fuzzy model into a 'crisp' model.

Algorithm:

Step 1: Solve the several objective assignment problems as a single objective assignment problem k times by taking one of the objectives at a time.

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Step 2: after step number 1, determine the corresponding values for every objective at each solution derived. According to each solution and value for every objective, we can find pay-off matrix as follows:

$$\begin{array}{c} \\ X^{(1)} \\ X^{(2)} \\ \vdots \\ X^{(k)} \end{array} \begin{array}{c} \begin{array}{cccc} Z_1(X) & Z_2(X) & \dots & Z_k(X) \end{array} \\ \begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1k} \\ Z_{21} & Z_{22} & \dots & Z_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{k1} & Z_{k2} & \dots & Z_{kk} \end{bmatrix} \end{array}$$

Where $X^{(1)}, X^{(2)}, \dots, X^{(k)}$ are the solutions as isolated optimal of the k different assignment problems for k
different objective functions. $Z_{ij} = Z_j(X^i)$ $(i=1,2,\dots,k \;\; \& \;\; j=1,2,\dots,k)$ be the i-th row and j-th column element of the pay-off matrix.

Step 3: in step 2, we find for each objective the worst (U_k) and the best (L_k) values corresponding to the set of solutions, where, $U_k = \max(Z_{1k}, Z_{2k}, \dots, Z_{kk})$ and $L_k = Z_{kk}$ $\quad k=1,2,\dots,K$

Step 4: by using linear and hyperbolic membership functions (μ or μ^H or exponential μ^E) for the k-th objective function as follows:

Case (i) using a linear membership function for the k-th objective function is defined by $\mu_k(X)$ and shown in Fig. (1).

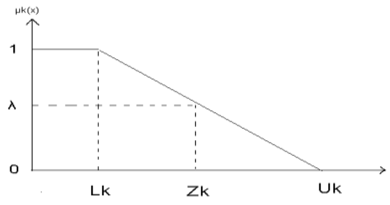

Fig. (1) The linear membership function

$$\mu_k(x) = \begin{cases} 1, & \text{if } Z_k \le L_k \\ 1 - \dfrac{Z_k - L_k}{U_k - L_k}, & \text{if } L_k < Z_k < U_k \\ 0, & \text{if } Z_k \ge U_k \end{cases} \tag{5}$$

Case (ii) using hyperbolic membership function for the k-th objective function is defined by $\mu^H_{Z_k}(x)$

$$\mu^H_{Z_k}(x) = \begin{cases} 1, & \text{if } Z_k \le L_k \\ \dfrac{1}{2}\,\dfrac{e^{\left\{\frac{(U_k+L_k)}{2} - Z_k(x)\right\}\alpha_k} - e^{-\left\{\frac{(U_k+L_k)}{2} - Z_k(x)\right\}\alpha_k}}{e^{\left\{\frac{(U_k+L_k)}{2} - Z_k(x)\right\}\alpha_k} + e^{-\left\{\frac{(U_k+L_k)}{2} - Z_k(x)\right\}\alpha_k}} + \dfrac{1}{2}, & \text{if } L_k < Z_k < U_k \\ 0, & \text{if } Z_k \ge U_k \end{cases} \tag{6}$$

Where, $\quad \alpha_k = 6/(U_k - L_k)$

Case (iii) using exponential membership function for the k^{th} objective function is defined by $\mu^E_{Z_k}(x)$

$$\mu^E_{Z_k}(x) = \begin{cases} 1, & \text{if } Z_k \le L_k \\ \dfrac{e^{-S\Psi_k(X)} - e^{-S}}{1 - e^{-S}}, & \text{if } L_k < Z_k < U_k \\ 0, & \text{if } Z_k \ge U_k \end{cases} \tag{7}$$

Where $\Psi_k(X) = \dfrac{Z_k - L_k}{U_k - L_k} \qquad k=1,2,\dots,K$

S is a non-zero parameter, prescribed by the decision maker.

Step 5: using step 4, we can find an similar crisp model for the initial fuzzy model as follows:

If we will use the linear membership function as defined in (5) then an equivalent crisp model for the fuzzy model can be formulated as:

Maximize λ

subject to

$$\lambda \le \frac{U_k - Z_k(X)}{U_k - L_k} \quad , k=1,2,\dots,K$$

$$\sum_{i=1}^{n} X_{ij} = 1, \qquad j=1,2,\dots,n\,; \quad \sum_{j=1}^{n} X_{ij} = 1, \qquad i=1,2,\dots,n\,; \quad \lambda \ge 0$$

$$X_{ij} = \begin{cases} 1, \text{ if the } i^{th} \text{ job is assigned to the } j^{th} \text{ machine} \\ 0, \text{ if the } i^{th} \text{ job is not assigned to the } j^{th} \text{ machine} \end{cases}$$

The above problem can be further simplified as:

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Maximize λ

subject to

$$Z_k(X) + \lambda(U_k - L_k) \le U_k \quad ,k=1,2,\ldots,K$$

$$\sum_{i=1}^{n} X_{ij} = 1, \quad j=1,2,\ldots,n; \quad \sum_{j=1}^{n} X_{ij} = 1, \quad i=1,2,\ldots,n; \quad \lambda \ge 0$$

$$X_{ij} = \begin{cases} 1, \text{ if the } i^{th} \text{ job is assigned to the } j^{th} \text{ machine} \\ 0, \text{ if the } i^{th} \text{ job is not assigned to the } j^{th} \text{ machine} \end{cases}$$

Using hyperbolic membership function as defined in equation (6) then an equivalent crisp model for the fuzzy model can be formulated as:

Maximize λ

subject to

$$\lambda \le \frac{1}{2}\,\frac{e^{\left\{\frac{(U_k+L_k)}{2}-Z_k(x)\right\}\alpha_k} - e^{-\left\{\frac{(U_k+L_k)}{2}-Z_k(x)\right\}\alpha_k}}{e^{\left\{\frac{(U_k+L_k)}{2}-Z_k(x)\right\}\alpha_k} + e^{-\left\{\frac{(U_k+L_k)}{2}-Z_k(x)\right\}\alpha_k}} + \frac{1}{2} \quad ,k=1,2,\ldots,K$$

$$\sum_{i=1}^{n} X_{ij} = 1, \quad j=1,2,\ldots,n; \quad \sum_{j=1}^{n} X_{ij} = 1, \quad i=1,2,\ldots,n; \quad \lambda \ge 0$$

$$X_{ij} = \begin{cases} 1, \text{ if the } i^{th} \text{ job is assigned to the } j^{th} \text{ machine} \\ 0, \text{ if the } i^{th} \text{ job is not assigned to the } j^{th} \text{ machine} \end{cases}$$

The above problem can be further simplified as

Maximize X_{nn+1}

subject to

$$\alpha_k\, Z_k(x) + X_{nn+1} \le \alpha_k (U_k + L_k)/2, \quad k = 1,2,\text{-----}K$$

$$\sum_{i=1}^{n} X_{ij} = 1, \quad j=1,2,\ldots,n; \quad \sum_{j=1}^{n} X_{ij} = 1, \quad i=1,2,\ldots,n; \quad X_{nn+1} \ge 0$$

$$X_{ij} = \begin{cases} 1, \text{ if the } i^{th} \text{ job is assigned to the } j^{th} \text{ machine} \\ 0, \text{ if the } i^{th} \text{ job is not assigned to the } j^{th} \text{ machine} \end{cases}$$

Where, $X_{nn+1} = \tanh^{-1}(2\lambda - 1)$

by using the exponential membership function as defined in (7) then an equivalent crisp model for the fuzzy model can be formulated as:

Maximize λ

subject to

$$\lambda \le \frac{e^{-S\Psi_k(X)} - e^{-S}}{1-e^{-S}} \quad ,k=1,2,\ldots,K$$

$$\sum_{i=1}^{n} X_{ij} = 1, \quad j=1,2,\ldots,n; \quad \sum_{j=1}^{n} X_{ij} = 1, \quad i=1,2,\ldots,n; \quad \lambda \ge 0$$

$$X_{ij} = \begin{cases} 1, \text{ if the } i^{th} \text{ job is assigned to the } j^{th} \text{ machine} \\ 0, \text{ if the } i^{th} \text{ job is not assigned to the } j^{th} \text{ machine} \end{cases}$$

The above problem can be further simplified as:

Maximize λ

subject to

$$e^{-S\Psi_k(X)} - (1-e^{-S})\lambda \ge e^{-S} \quad ,k=1,2,\ldots,K$$

$$\sum_{i=1}^{n} X_{ij} = 1, \quad j=1,2,\ldots,n; \quad \sum_{j=1}^{n} X_{ij} = 1, \quad i=1,2,\ldots,n; \quad \lambda \ge 0$$

$$X_{ij} = \begin{cases} 1, \text{ if the } i^{th} \text{ job is assigned to the } j^{th} \text{ machine} \\ 0, \text{ if the } i^{th} \text{ job is not assigned to the } j^{th} \text{ machine} \end{cases}$$

Step 6: Solving the crisp model by an appropriate mathematical programming algorithm.

The solution obtained in step 6 will be the optimal compromise solution of the Multi-objective assignment problem

Numerical Example

Minimize: $Z_1 = 10X_{11}+8X_{12}+15X_{13}+13X_{21}+12X_{22}+13X_{23}+8X_{31}+10X_{32}+9X_{33}$

Minimize: $Z_2 = 13X_{11}+15X_{12}+8X_{13}+10X_{21}+20X_{22}+12X_{23}+15X_{31}+10X_{32}+12X_{33}$

Subject to

$$\sum_{i=1}^{3} X_{ij} = 1, \quad j=1,2,3; \quad \sum_{j=1}^{3} X_{ij} = 1, \quad i=1,2,3$$

$$X_{ij} = \begin{cases} 1, \text{ if the } i^{th} \text{ job is assigned to the } j^{th} \text{ machine} \\ 0, \text{ if the } i^{th} \text{ job is not assigned to the } j^{th} \text{ machine} \end{cases}$$

For the objective Z_1, we find the optimal solution as

$$X^{(1)} = \begin{cases} X_{12}=1,\ X_{23}=1,\ X_{31}=1, \\ \text{and rest all } X_{ij}\text{'s are zeros} \end{cases}$$

and $Z_1 = 29$

For the objective Z_2, we find the optimal solution as

$$X^{(2)} = \begin{cases} X_{13}=1,\ X_{21}=1,\ X_{32}=1, \\ \text{and rest all } X_{ij}\text{'s are zeros} \end{cases} \quad \text{and}$$

$Z_2 = 28$

We can write the payoff matrix as

$$\begin{array}{c c} & \begin{array}{cc} Z_1(X) & Z_2(X) \end{array} \\ \begin{array}{c} X^{(1)} \\ X^{(2)} \end{array} & \begin{bmatrix} 29 & 38 \\ 42 & 28 \end{bmatrix} \end{array}$$

From the pay-off matrix we find the upper bound and lower bound

U_1 = max (29, 38) = 38, U_2 = max (42, 28) = 42, L_1 = 29, L_2 = 28, d_1 = 9, d_2 = 14

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If we use the linear membership function as defined in(5), an equivalent crisp model can be formulated as:

$$\text{Maximize} \quad \lambda$$

Subject to

$$10X_{11}+8X_{12}+15X_{13}+13X_{21}+12X_{22}+13X_{23}+8X_{31}+10X_{32}+9X_{33}+9\lambda \le 38$$

$$13X_{11}+15X_{12}+8X_{13}+10X_{21}+20X_{22}+12X_{23}+15X_{31}+10X_{32}+12X_{33}+14\lambda \le 42$$

$$\sum_{i=1}^{3} X_{ij}=1, \quad j=1,2,3\,; \sum_{j=1}^{3} X_{ij}=1, \quad i=1,2,3\,;\ \lambda \ge 0$$

$$X_{ij}=\begin{cases}1, \text{ if the } i^{th} \text{ job is assigned to the } j^{th} \text{ machine}\\ 0, \text{ if the } i^{th} \text{ job is not assigned to the } j^{th} \text{ machine}\end{cases}$$

The problem is solved by the linear interactive and discrete optimization (LINDO) software. The optimal solution is presented as follows:

$$X^{*} = \begin{cases} X_{12}=1,\ X_{21}=1,\ X_{33}=1, \\ \text{and rest all } X_{ij} \text{ 's are zeros}\end{cases}$$

$$Z_1^{*}=30,\ Z_2^{*}=37 \text{ and } \lambda = 0.58$$

If we use the hyperbolic membership function as defined in (6), an equivalent crisp model can be formulated as:

$$\text{Maximize} \quad X_{10}$$

Subject to

$$60X_{11}+48X_{12}+90X_{13}+78X_{21}+72X_{22}+78X_{23}+48X_{31}+60X_{32}+54X_{33}+9X_{nn+1} \le 201$$

$$78X_{11}+90X_{12}+48X_{13}+60X_{21}+120X_{22}+72X_{23}+90X_{31}+60X_{32}+72X_{33}+14X_{nn+1} \le 210$$

$$\sum_{i=1}^{3} X_{ij}=1, \quad j=1,2,3\,; \sum_{j=1}^{3} X_{ij}=1, \quad i=1,2,3\,;\ X_{10} \ge 0$$

$$X_{ij}=\begin{cases}1, \text{ if the } i^{th} \text{ job is assigned to the } j^{th} \text{ machine}\\ 0, \text{ if the } i^{th} \text{ job is not assigned to the } j^{th} \text{ machine}\end{cases}$$

The problem is solved by the linear interactive and discrete optimization (LINDO) software. The optimal solution is presented as follows:

$$X^{*} = \begin{cases} X_{12}=1,\ X_{21}=1,\ X_{33}=1, \\ \text{and rest all } X_{ij} \text{ 's are zeros}\end{cases}$$

$X_{nn+1}=0.4818653$

But here,

$X_{nn+1}=\tanh^{-1}(2\lambda-1)$

$\tanh(0.4818653)= 2\lambda-1$

$\lambda=0.50$

Therefore

$$Z_1^{*}=30,\ Z_2^{*}=37 \text{ and } \lambda = 0.50$$

However, If we use exponential membership function as defined in (7) with the parameter S=1, an equivalent crisp model for the fuzzy model can be formulated as:

$$\text{Maximize} \quad \lambda$$

Subject to

$$\exp\{(-10X_{11}-8X_{12}-15X_{13}-13X_{21}-12X_{22}-13X_{23}-8X_{31}-10X_{32}-9X_{33}+29)/9\}$$
$$-0.6321205\lambda \ge 0.3678794$$

$$\exp\{(-13X_{11}-15X_{12}-8X_{13}-10X_{21}-20X_{22}-12X_{23}-15X_{31}-10X_{32}-12X_{33}+28)/14\}$$
$$-0.6321205\lambda \ge 0.3678794$$

$$\sum_{i=1}^{3} X_{ij}=1, \quad j=1,2,3\,; \sum_{j=1}^{3} X_{ij}=1, \quad i=1,2,3\,;\ \lambda \ge 0$$

$$X_{ij}=\begin{cases}1, \text{ if the } i^{th} \text{ job is assigned to the } j^{th} \text{ machine}\\ 0, \text{ if the } i^{th} \text{ job is not assigned to the } j^{th} \text{ machine}\end{cases}$$

The problem is solved by using GINO software. The optimal solution is presented as follows:

$$X^{*} = \begin{cases} X_{12}=1,\ X_{21}=1,\ X_{33}=1, \\ \text{and rest all } X_{ij} \text{ 's are zeros}\end{cases}$$

$$Z_1^{*}=30,\ Z_2^{*}=37 \text{ and } \lambda = 0.45$$

CONCLUSIONS :

In this paper, to solve the multi-objective assignment problem using linear and non-linear membership functions have been used. The crisp model becomes linear If we use the hyperbolic membership function. The optimal compromise solution does not change significantly if we compare with the solution obtained by the linear membership function. Fuzzy several objective linear programming problem in which both the resources and the technological coefficients are fuzzy in nature. Further a FMLOP problem was converted into an equivalent crisp non-linear programming problem using the concept of principal of max-min. The resultant non-linear programming problem was solved by fuzzy decisive set method. The above explained method was illustrated by an example. In future proposed method can be extended to solve problems like FMLOP with various membership function and linear fuzzy fractional programming problems. Stochastic uncertainty relates to the uncertainty of occurrences of phenomena or events. Its characteristics, lie in that descriptions of information are crisp and well defined; however, they vary in their frequency of occurrence. The

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systems with this type of uncertainty are called stochastic systems, which can be solved by stochastic optimization techniques using probability theory. However, if we use the other than linear type membership function, with different values of parameter then the crisp model becomes non-linear and the optimal compromise solution does not change significantly, if we compare with the solution obtained by the linear membership function.

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